The effect of climate variations on the dynamics of pasture–livestock interactions under cooperative and noncooperative management

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It is well known from Hardin's "Tragedy of the Commons" [Hardin G (1968) *Science* 162:1243–1248] that, if open access is allowed, overgrazing typically results. Hardin, and most authors of the subsequent literature, adopted a static view of the underlying ecosystem. Here we extend this tragedy of the commons to consider the dynamics of the involved ecosystem as well. We consider a general model that allows for a variable carrying capacity of the pastures (due to variation in precipitation) and a stimulating effect on plant growth due to grazing. Our analysis further emphasizes the tragedy; in addition to overgrazing, the ecosystem may approach limit cycles. Thus, unless the pastoralists are able to coordinate themselves, the human capability of long-term planning will generally not stabilize the system. Although numerical optimization shows that a cooperative optimum would yield a high and stable harvest, the open-access system may produce limit cycles, in which even the peak harvest may be below the stable cooperative optimal harvest. Such fluctuations cause both losses in biomass production and utility losses. Our dynamic analysis also demonstrates that, in the absence of cooperation between herders, too much rain in an otherwise dry area might (temporally) destabilize the ecological grazing system through overstocking, subsequently leading to further overgrazing (which will be observed in, but not caused by, the typically dry conditions of landscapes where pastoralism is practiced). In short, through this study we have brought time (and temporal dynamics) into the Hardin's tragedy of the commons and show that the tragedy might be profoundly worsened.

ecological dynamics | pastoralism | resource management | tragedy of the commons

Hardin's argument (1) may be, and indeed has been, criticized for confusing open access and common property, and disregarding informal institutions governing common properties (see, for example, refs. 2–5). Hardin's analysis also ignored the dynamics of the underlying ecosystem and the roles played by the stochastic precipitation that drive biomass production in the dry lands (6). Still, Hardin's argument is an important reminder of the potential dangers when appropriate management institutions fail. Hardin's argument is similar to what Sinclair and Fryxell (7) also referred to as the "settlement and overgrazing hypothesis" in reference to the Sahelian rangelands.

Although Hardin's original analysis and most of the subsequent literature relied on a static description of the underlying ecosystem, we will add ecological dynamics to the analysis. For this, we use a standard dynamic model for tropic interaction (8), but we add a climate-variation component. Incorporation of the intrinsically caused dynamics (due to ecological self-regulation) and extrinsically caused dynamics (e.g., due to climatic variation) of the ecosystem reveals another dimension to the "tragedy" (see, for example, ref. 4): the loss of productivity of the ecosystem due to fluctuations. The temporal dimension introduces a need for intertemporal coordination among the herders. Although human perception of the dynamics of such a system is, at best, imperfect (9, 10), we consider the case in which humans, through their ability to plan long term, are most likely to be able to avoid instability (that is, humans have perfect foresight and an infinite planning horizon). Our analysis shows the need for intertemporal coordination to avoid instability.

Materials and Methods

We study the system under two different conditions. A formal description of the two cases is given below. The first case involves farmers who are unable to coordinate. To simplify the analysis, we then assume that the herd of each farmer is so small, relative to the total biomass of animals grazing on the commons, that the individual farmer considers his impact on the grazing land as negligible. Hence, the individual farmer will optimize consumption subject to the constraint

$$\dot{y}_{it} = (h\phi(X_t) - m)y_{it} - c_{it} = a_t y_{it} - c_{it_D},$$

where X_t is given and is beyond the herders' control. Note that the optimal consumption strategy may depend on the path of abundance of vegetation, but only through its effect on the path of animal growth rate, a_t.

To simplify further, we assume that all herders have the same utility function, and hence all will follow the same consumption strategy. Within the class of utility function that we consider, it turns out that the optimal strategy will be to consume a share λ_t of the total livestock, where λ_t may depend only on the path of a_t. Hence, total consumption C_t is a function of X_t and Y_t, and the dynamics of the ecosystem as specified above is well defined.

The second case we study involves herders who are able to coordinate. To simplify further, we now assume that the herders not only have equal utility functions but also have herds of equal size initially. This assumption avoids issues of income distribution in the definition of the cooperative solution. When all herders are equal, they all consume an equal share of total consumption, and hence the question is to choose the path of C_t to maximize each farmer's utility. Because this is an optimal control problem with two dynamic state variables, we have been unable to derive an analytical solution and have reverted to numerical optimization.

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Results and Discussion

The analysis shows that, although the ecosystem would be stable under cooperative management, overgrazing and instability emerge if herders are unable to cooperate and control free riding. In both cases, we consider common property regimes in which a fixed number of herders have exclusive rights to use a common grazing area. Thus, the tragedy is not a tragedy of the commons but one of failing institutions. Indeed, there are abundant examples of common properties that are successfully managed (4, 11), but there are also examples where such institutions fail due to pressure from population growth or other changes. It is well known that pastoral societies undergo great changes today implying among other things that traditional management institutions (such as the elders) are being replaced by more individual-based enterprises (12). The world's climate is also changing; more extreme weather conditions are likely to occur more frequently (13). Here we analyze the dynamics effects of both different management regimes (the ability to coordinate) and climate variation within a terrestrial system exemplified by an arid grazing-land pastoral system. Note also that terrestrial systems such as the one in our study are different from marine systems, for which the impact of human planning has been widely studied (14), because those exploiting marine systems do not own individual fish stocks like pastoralists own their herds of grazing animals.

The dynamic properties of such a plant-grazer system, when neither plants nor animals are managed, is well known within the field of ecology; such a system might easily exhibit periodic fluctuations like limit cycles (15–17), dynamic behavior which is also well known within the field of economics (see, for example, refs. 18 and 19). Less studied (but see ref. 14) is the impact of human management on an ecological system in which oscillations may recur. Fluctuations are likely to reduce the overall productivity of the ecological system. In addition, fluctuating consumption represents a separate utility loss. But will the foresight of rational people avoid the cycles that often occur in the unmanaged system?

We include several modifications of the typical textbook plant-grazer model: (*i*) We consider biomass rather than the number of plants and animals. This change seems appropriate for two reasons. First, pasture quality is typically measured in biomass density. Second, we will be able to consider the combined effect of death and reduction in weight (or reproduction and fattening). (*ii*) We consider a varying carrying capacity that allows for a nonequilibrium perspective (see, for example, ref. 6): with occasional deviations from the average level of precipitation. (*iii*) Finally, we incorporate a positive effect of grazing on plant regrowth (20). We use the following notation: for the grazing animals, Y and y_i represent, respectively, the total biomass abundance of grazing animals on the pastures and the biomass abundance (animals) belonging to herder i (i.e., $Y = \sum y_i$, where the sum is over the fixed number of herders with rights to use the grazing area). Because the pasture is common property, the biomass abundance (vegetation) of the pasture is given by X.

Consider an individualistic herder keeping livestock y_t with growth rate a_t and a harvesting rate, c_t, describing the rate at which livestock is slaughtered for consumption or sale. We assume that the herder chooses harvest so as to maximize

$$\int_0^\infty \ln(c_t)e^{-\delta t}dt,$$

where δ is the utility discount rate. The path of growth rate, a_t, is determined by the abundance of plants on the pasture, but each individual herder only has a small impact on the vegetation, and hence we assume that the herder takes a_t as given and beyond his control. Although plant abundance is beyond the herders' control, we assume that the herders fully understand the dynamics of the ecosystem. Indeed, we assume that the herders have an infinite time horizon. Furthermore, although not essential to the conclusion, we also assume that the pastoralists have perfect foresight in the sense that the growth rate, a_t, which may essentially be any stochastic process, is fully known. The herder is fully rational in the sense of being able to choose the harvest rate that maximizes utility even in stochastic conditions. As shown in supporting information (SI) *Text*, the optimal harvest will be a constant fraction of the biomass of animals, given by $c_t = \delta y_t$, where δ is the utility discount rate. Given that herders use the optimal harvest strategy, the dynamics of the livestock-biomass for a single herder will be given as

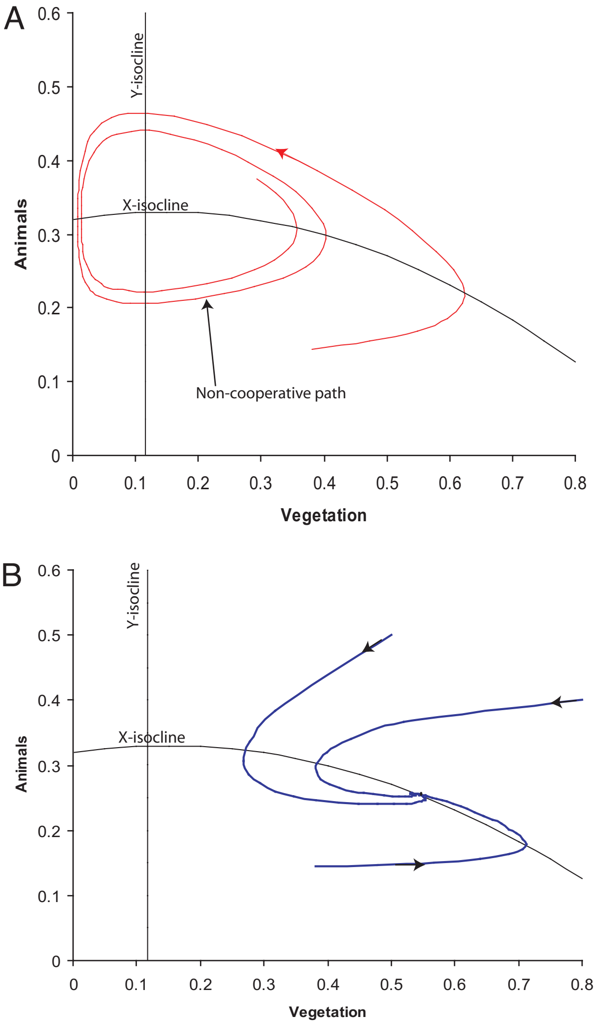

Fig. 1. Noncooperative (*A*) and cooperative (*B*) solution. This figure and the following figures are all based on the specification $h' = 10$; $h = 1{,}5$; $\delta = 10\%$; $m = 10\%$; $\phi(X) = X/(X + 0.7)$; and $g(Y) = 3 + 2Y/(Y + 0.1)$ (see *SI Text* for choice of parameter values). In Fig. 1 we also have $K = 1$. The dynamic paths correspond to the cases with (*B*) and without (*A*) coordination. The *X*-isocline corresponds to $K = 1$. The intersection of the zero-growth line for the livestock (the *Y*-isocline) and the zero-growth line for the pasture-vegetation (the *X*-isocline) represents the ecological equilibrium, an equilibrium that is unstable if the *Y*-isocline crosses the *X*-isoclines to the left of the hump of the *X*-isoclines (as in this case) and stable if the *Y*-isoclines crosses the *X*-isoclines to the right of this hump. The path converging to the limit cycle corresponds to the uncoordinated path, whereas the path converging to a high level of vegetation is the optimal cooperative solution. We also show other convergence paths for the optimum, with different initial states.

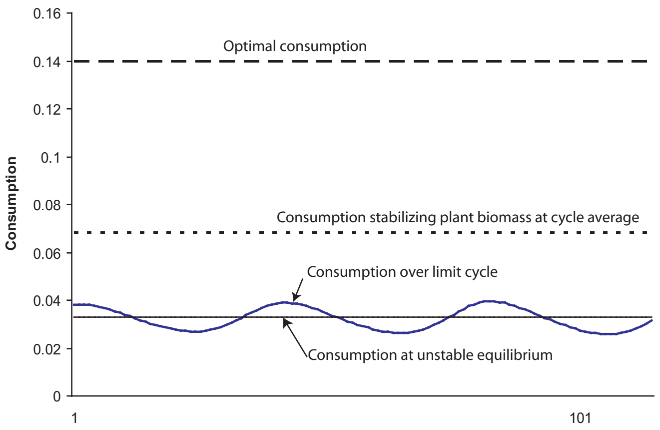

Fig. 2. Consumption. The solid line is the noncooperative solution, whereas the high dashed line is the cooperative solution, in both cases after the system has been given time to converge to the limit cycle or steady state, respectively. The dotted line represents the consumption that would prevail if the system had been stabilized at a level of plant biomass equal to the average over the limit cycle, whereas the weakly drawn line is the consumption when the system is in the unstable equilibrium.

$\dot{y} = -(m + \delta)y + hy\phi(X)$, where $\phi(X) = X/(X + \eta)$ (where η is the half-saturation constant) is the functional response curve linking grazing to the abundance of plants on the pasture (cf. ref. 21), h is the maximum consumption rate, and m is some natural mortality (or biomass reduction) rate for the livestock. Because the right-hand side of this dynamic model is linear in y, the dynamics immediately carries over to the aggregate livestock for all herders. It follows that we can write $\dot{Y} = -(m + \delta)Y + hY\phi(X)$ for the livestock by using the considered pasture; as can be seen, human harvest only adds to the fixed mortality rate. In absence of grazing livestock, the plant population is assumed to follow the logistic equation (8). Adding grazing livestock, the plant population dynamics is given by $\dot{X} = g(Y)(K - X)X - Y\phi(Y)$, where K is carrying capacity (which we will assume to be affected by the level of precipitation), and the g-function incorporates the effect of grazing on plant growth (20). The optimal harvest strategy (i.e., the slaughtering of livestock) $c_t = \delta y_t$ applies with stochastic livestock growth rates, and hence the optimal solution of the harvesting problem will not be affected by whether K is fixed or not.

Combining the above models for the livestock and the pasture, we obtain the following ecological model of the pasture-livestock system:

$$\dot{X} = g(Y)(K - X)X - h'Y\phi(X)$$

$$\dot{Y} = -(m + \delta)Y + hY\phi(X).$$

As is well known from ecology (see, for example, chap. 10.2 in ref. 8; see also ref. 16), such a system may exhibit limit cycles, especially when the vegetation equilibrium without grazing livestock is high (as might be the case after much rain) and/or if the efficiency of the grazing livestock in consuming vegetation is high.

In *SI Text*, we compare the optimal management under noncooperative and cooperative management. We have solved the cooperative system numerically for given parameter values, but in all simulations the system quickly converges to a steady state. For this result, the assumption of perfect foresight is important (see ref. 10). We also show that a fluctuating system is less productive, and hence we would intuitively expect the optimal cooperative solution to be stable.

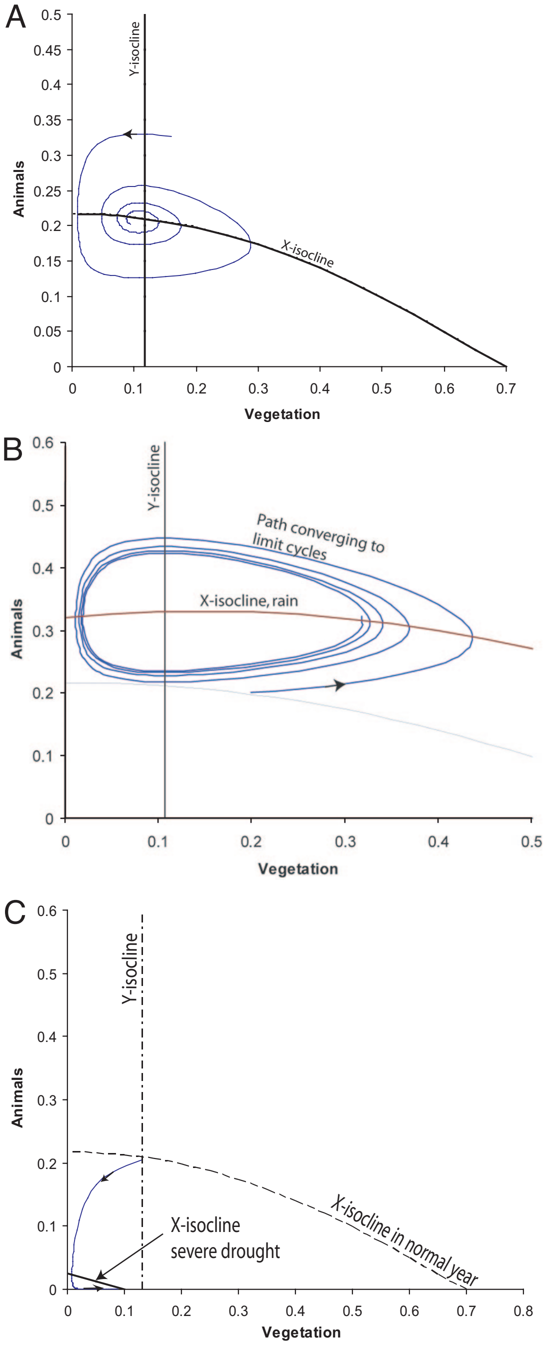

Fig. 3. The dynamic effects of changing environmental conditions. (*A*) The "typical" equilibrium condition. In this case the equilibrium is stable. Occasional low levels of precipitation will lead to a lower equilibrium level of livestock, and potentially to extinction of animals (*C*), whereas occasional heavy precipitation may bring the pasture–livestock system into ecological cycles resulting in a low level of vegetation following years of heavy precipitation (*B*).

Fig. 1 illustrates the dynamic path both with and without cooperation. The convergence paths, *B*, are those chosen by a hypothetical planner (or elders managing the common pastures) and are referred to as the cooperative solution, whereas the limit cycle is the noncooperative path that will emerge without the planner. Note that the level of plant in the steady state (the point the convergence paths converges to) is much higher than the average of the limit cycle, which is the traditional tragedy of the

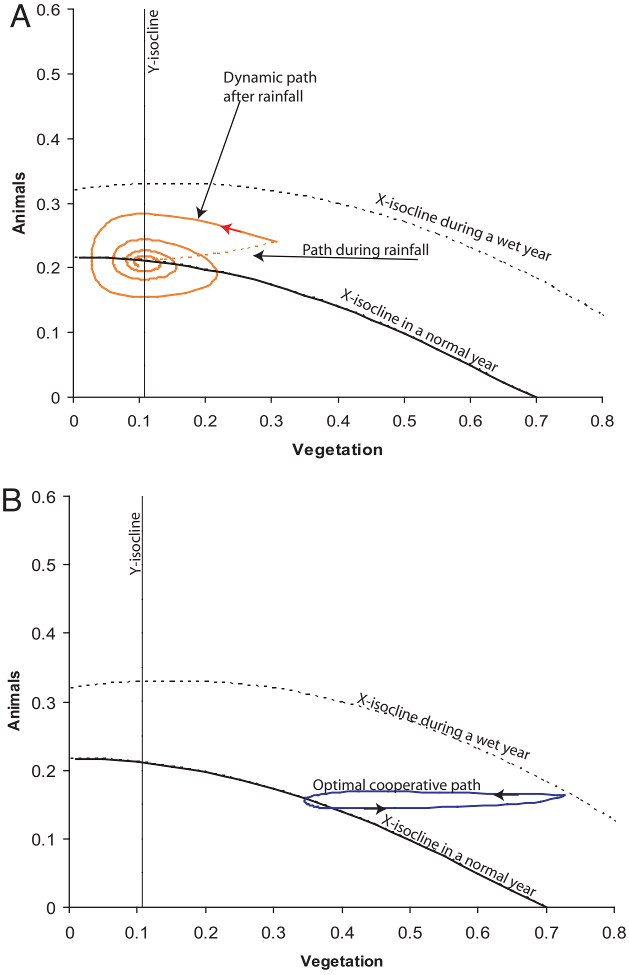

Fig. 4. Noncooperative (*A*) and cooperative (*B*) response to a 1-year increase in precipitation. The solid *X*-isocline represents the typical condition, whereas the dashed line represents years with high precipitation. The weakly drawn path shows the dynamic path during a year of high precipitation. The solid dynamic path then illustrates the development after a year of high precipitation, where the system slowly converges back to the original equilibrium. The cooperative optimal path starts out at a higher level of vegetation that represents the steady state for this cooperative system with low vegetation. Provided the planner knows that the high level of precipitation will last exactly 1 year, the path shown is the optimal path in response to the increased rainfall.

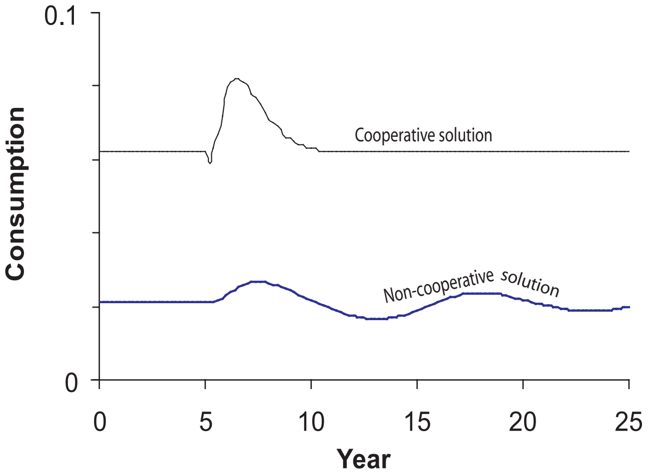

Fig. 5. Consumption with and without cooperation. The rainfall starts at year 5 and lasts for 1 year. Note here that in the optimal path, the consumption initially drops to a lower level to allow a more rapid increase in the biomass of livestock able to exploit the abundance of vegetation. The optimal path quickly converges to a new steady state, whereas the noncooperative path shows damped oscillations.

commons, and the steady-state harvest rate with the given parameters is more than 6 times as high. The striking difference in stability under cooperation and noncooperation is worth noticing. Note further that the figures show the convergence path for different initial stocks of plants and livestock, but all quickly converge to the same steady state. Fig. 2 shows the corresponding consumption paths. Most importantly, the harvest is higher than the peaks of the noncooperative solution. Thus with the animal growth explicitly modeled, Hardin's claim that farmers put too many animals on the grazing field translates into a claim that farmers use a too low harvest rate.

As an assessment of the cost of cycles, we may compare the above results (Fig. 2) with the steady state of the ecosystem (where the two isoclines intersect). The cycle in Fig. 2 amounts to an approximate ±18% deviation in livestock from steady state, which yields a small utility loss corresponding to a permanent 1% loss in harvests. If animal growth responds more strongly to plant abundance, losses will be much higher; in *SI Text*, we provide examples in which the losses correspond to a permanent 24% reduction in harvest due to fluctuations compared with the steady-state condition. It should be noticed that the steady state depends on the dynamics of the system and is different from what is expected from a static nondynamic model (see *SI Text*).

When adding ecological dynamics to Hardin's static analysis, we thus see that the lack of coordination not only causes overgrazing but may also cause extensive ecological fluctuations that intensify the tragedy. Indeed, the tragedy of the commons was suggested as an explanation of the Sahelian famine of the 1970s (see ref. 22), and experiences from the Sahelian area show some resemblance with ecological limit cycles. During periods of surplus plant cover, the herds increase, but during periods with shortage of vegetation for the livestock, animals die off.

The basic results are maintained if we allow pastoralists to have preferences for large stocks of animals as symbols of status, only now the optimal harvest rate is lower, which is worsening the problem. We further argue that within a reasonable class of utility functions, a more concave function is likely to yield more fluctuations, whereas a less concave function is likely to yield more stability. Finally, we consider the case in which animals can be bought and sold at a fixed price from outside markets and with perfect credit markets. In this case, financial wealth and animals are alternative means of storing values, and the only possible equilibrium is a level of plants where animals and financial wealth yield the same rate of return, if pastoralism is at all economically viable. Even in this case, there is overgrazing compared with the cooperative solution.

Thus far, we have considered only the deterministic case. However, the theorem given in *SI Text* shows that the harvest strategy $c_t = \delta y_t$ is individually optimal for almost any stochastic animal growth rate. We will only consider a case in which the carrying capacity K may take different values, depending on the amount of rainfall. Fig. 3 provides some intuitive insight into the dynamic effect of varying levels of occasional high or low levels of precipitation. Fig. 4 illustrates the case in which K initially is low, and the ecosystem is stable. A period of rainfall increases K and shifts the isocline upward and to the right, making the ecosystem unstable. The ecosystem enters into a cycle. Eventually, the effect of rainfall ends, and the ecosystem drops back to the original dynamics, but because of previous overgrazing, the system takes a long time to recover. However, it is the increased level of precipitation, and not the draught, that has caused the dynamically unfavorable conditions.

For the simulations in Fig. 4, it makes no difference if the start or end of rainfall is predicted, because the harvest rate is the same in all cases. The rain period is assumed to last 1 year, but the picture will be similar for longer rain periods. If the rain period lasts for a long period, the system (with the given parameters) will converge to limit cycles, but as the rain stops, it will not drop back to the stable equilibrium of the dry conditions but will converge through damped oscillations.

Fig. 4 also presents the optimal cooperative path starting in the steady state of the cooperative system, assuming that the onset of rain is a surprise but the end in 1 year is predicted. In this case, expectations about future rainfall do have an effect. If the rain was expected to be permanent, the optimal path would aim for a steady state with significantly more livestock (an increase by almost 100% from 0.14 to 0.25), and the initial harvest would have been somewhat lower to allow a faster growth of livestock. Notice that the initial fluctuations are due to the ecological dynamic interactions, whereas the following dynamics and approach to stability is chosen by the management regime.

The consumption path in this simulation is shown in Fig. 5. Rains starts after 5 years and lasts for exactly 1 year. The cooperative path shows a peaked curve, enjoying increased consumption during rainfall and for some time after the end of the rainfall but quickly converging back to the steady state.

For the noncooperative solution, the figure illustrates that there is, of course, an initial benefit from the rainfall, but eventually the cost of overgrazing appears. Consumption fluctuates for a long period until the system recovers. Whether the short-run benefits outweigh the long-run cost depends on the discount rate. This particular simulation presumes that $m+\delta = 20\%$, and we find that for $\delta < 5\%$, rainfall is actually harmful to the pastoralists, as discounted utility is lower in the rainfall scenario, whereas with cooperation the rainfall would yield a increased utility of the same size as a 4% permanent increase in harvests. That is, when we observe overgrazing, it might well be due to very good pastures (resulting form some occasional good rain) at an earlier instance (e.g., the previous year) and not due to the drought as such.

Thus, our analysis indicates that the lack of adequate institutions may have particularly severe consequences when herders face shorter periods of good luck ("good rainfall"). Without institutions to ensure cooperation, a year of rainfall that should have been a blessing may become a curse. Because of climate change (13), this might be more of a problem in the future than is currently appreciated.

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